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1.0 Derivation of the basic equation

The fundamental principles upon which these averaging pitot tubes operator are utilizing the Bernoulli relationship. Referring to Figure 1, the total or impact pressure on the upstream side of the tube when in an incompressible fluid may be obtained from the Bernoulli relationship as:

$$P_{F2} = P_{F1} + \frac{\rho_1 V_1^2}{2g_c} \tag{1.1}$$

Where P_1 and V_1 refer to the pressure and velocity upstream from the pressure port. P_1 is called the static or free stream pressure and $\frac{1}{2}\rho_1 V_1^2$ is called the dynamic or stagnation pressure. The flow around the tube causes part of the dynamic pressure to be subtracted from the static pressure sensed so that the downstream opening sees a pressure of:

$$P_{F3} = P_{F1} - \frac{C\rho_1 V_1^2}{2g_c} \tag{1.2}$$

Where C is a constant that can be determined experimentally.

$$DP = P_{F2} - P_{F3} = (1 + C)\frac{\rho_1 V_1^2}{2g_c} \tag{1.3}$$

Letting $1 + C = B$, this equation can be arranged to give:

$$V = \sqrt{\frac{2g_c DP}{B\rho_1}} \tag{1.4}$$

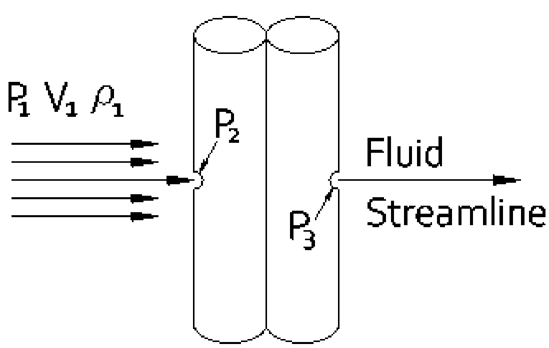

Figure 1:

Strictly speaking, the velocity calculated from equation 1.4 is that of the fluid on a streamline directly upstream from the upstream port.

In normal pipe flow that is well developed, a velocity profile exists which will cause the differential pressure measured by such a device to vary with location within the pipe. The multiple pressure ports of the sensor are located at the proper position in normal pipe flow to sense a differential pressure which will be proportional to the average or mean velocity of the fluid in a pipe. The average velocity is therefore produced when using equation 1.4 and DP is the differential pressure produced by the multi port sensor. The volume flow rate may be obtained from the continuity equation as:

$$Q = AV = A\sqrt{\frac{2g_c DP}{B\rho_1}} \tag{1.5}$$

Deviations from ideal theory and the constant B may be combined into a single flow coefficient K which results:

$$Q = KA\sqrt{\frac{2g_c DP}{B\rho_1}} \tag{1.6}$$

For conditions where acceleration due to gravity is equal to the gravitational constant of proportionality of 32.174 (a good approximation for any place on the earth's surface) pounds force (Lb_f) is equal to pounds mass (Lb_m) and g_c becomes g. Therefore:

$$\frac{DP(\frac{lbs}{ft^2})}{\rho_1(\frac{lb}{ft^3})} = h(ft) \tag{1.7}$$

Where h is equal to the height of a pressure column of the flowing fluid in feet. Substituting eq. 1.7 into 1.6 gives the basic equation.

$$Q = KA\sqrt{2gh} \tag{1.8}$$

2.0 Formulas for use with water or any liquid

For liquid the convenient units for flow is Gal(U.S. Liq.)/Min or GPM. From equation 1.8:

$$Q_{Lf}(GPM) = \frac{K\pi {D_i}^2}{4}\sqrt{\frac{2(32.1741 ft/sec^2)\ \Delta P\ (3.60916\times 10^{-2}\ \frac{lb/in^2}{inW.C.})}{(62.3707\ \frac{lb}{ft^3})\ S_f}\left[\frac{60\ sec/min\ (7.48052\ gal/ft^3)}{144\ in^2/ft^2}\right]} \tag{2.1}$$

$$Q_{Lf}(GPM) = 5.66856\ K\ {D_i}^2\sqrt{\frac{\Delta P}{S_f}} \tag{2.1}$$

The volumetric flow rates measured under flowing conditions can be converted to base conditions (60°F and 14.6959 psia) by multiplying by a ratio of specific gravities. From the principle that mass flow rates always remain equal in a steady state system:

$$M = Q_{Lf}\cdot\rho_f = Q_{Ls}\cdot\rho_s \tag{2.2}$$

and solving for Q_{Ls} we have:

$$Q_{Ls} = Q_{Lf}\ \frac{\rho_f}{\rho_s} = Q_{Lf}\ \frac{S_f}{S_s} \tag{2.3}$$

Then substituting equation 2.3 into 2.1 results:

$$Q_{Ls} = \frac{5.66856\ K\ {D_i}^2}{S_s}\ \sqrt{\Delta P\ S_f} \tag{2.4}$$

or:

$$\Delta P = \frac{{Q_{Ls}}^2\ {S_s}^2}{S_f\ K^2\ {D_i}^4\ (32.1325)} \tag{2.5}$$

The rearranged version of equation 2.1 is:

$$\Delta P = \frac{{Q_{Lf}}^2\ S_f}{K^2\ {D_i}^4\ (32.1325)} \tag{2.6}$$

3.0 Formulas for use with any gas

For gas flow applications the standard units are actual cubic feet per minute (ACFM), and standard cubic feet per minute (SCFM). ACFM is the volumetric flow rate at flowing conditions, SCFM is the volumetric flow rate that would be measured if the same mass flow rate were occurring under standard conditions. Standard conditions are defined here as 14.6959 psia and 60°F (519.67°R).

$$Q_a = K\left[\frac{\pi {D_i}^2 in^2}{4}\right]\left[\frac{ft^2}{144\ in^2}\right]\left[\frac{60\ sec}{min}\right]\sqrt{\frac{2(32.1741\ \frac{ft}{s^2})\ \Delta P\ (3.60916\times 10^{-2}\ \frac{lb/in^2}{inW.C.})(144\ \frac{in^2}{ft^2})}{\rho_f}} \tag{3.1}$$

$$= 5.98455 K {D_i}^2\sqrt{\frac{\Delta P}{\rho_f}} \tag{3.1}$$

Or, Since by the ideal gas law:

$$\rho_f = \rho_{air}\ S_s\ \frac{P_f}{P_s}\ \frac{T_s}{T_f} \tag{3.2}$$

$$Q_A = 5.98455 K {D_i}^2 \sqrt{\frac{\Delta P\ (14.6959\ lb/in^2)\ T_f}{(7.6355E^{-2}\ lb/ft^3)\ P_f\ (519.67^\circ R)\ S_s}} \tag{3.3}$$

$$= 3.64206 K {D_i}^2 \sqrt{\frac{\Delta P\ (t_F + 459.67)}{P_f\ S_s}} \tag{3.3}$$

To obtain the flow rate in SCFM the ideal gas law is again used to convert ACFM. Since:

$$\frac{P_f\ Q_a}{T_f} = \frac{P_s\ Q_s}{T_s} \tag{3.4}$$

We have

$$Q_s = \frac{P_f\ Q_a\ T_s}{P_s\ T_f} \tag{3.5}$$

Incorporating equation 3.5 into 3.3:

$$Q_s = 3.64206\ K\ {D_i}^2\ \frac{T_s}{P_s} \sqrt{\frac{\Delta P\ T_f}{P_f\ S_s}\ \left[\frac{{P_f}^2}{{T_f}^2}\right]} \tag{3.6}$$

$$Q_s = 128.789\ K\ {D_i}^2\ \sqrt{\frac{\Delta P\ P_f}{(t_f + 459.67)\ S_s}} \tag{3.6}$$

Or:

$$\Delta\ P = \frac{{Q_s}^2\ S_s\ (t_f + 459.67)}{K^2\ {D_i}^4\ P_f\ (16,586.6)} \tag{3.7}$$

The rearranged version of equation 3.3 is:

$$\Delta\ P = \frac{{Q_a}^2\ P_f\ S_s}{K^2\ {D_i}^4\ (t_f + 459.67)\ (13.2646)} \tag{3.8}$$

4.0 Formulas for use with steam or any gas

In the case of steam and some other gases the convenient units of flow are often pounds per hour (PPH). To obtain these units a density factor is added to the ACFM equation (3.1)

$$Q_p = Q_a\ \rho_f \tag{4.1}$$

$$Q_p = 5.98455\ K\ {D_i}^2\ \frac{60\ min}{hr} \sqrt{\frac{{\rho_f}^2\ \Delta P}{\rho_f}} \tag{4.1}$$

$$Q_p = 359.073\ K\ {D_i}^2\ \sqrt{\Delta P\ \rho_f} \tag{4.2}$$

Or,

$$\Delta P = \frac{{Q_p}^2}{K^2\ {D_i}^4\ \rho_f\ (128933)} \tag{4.3}$$

Equations 4.2 or 4.3 may be used on any flowing gas.

1 Definition and units of measurement for symbols & references

A - Area ft^2

cp - Absolute viscosity in centipoise

Subscript s - Value when at standard conditions (60 °F, 14.6959 psia)

Subscript f - Value when at flowing conditions

D_i - Pipe inside diameter (inch) for square and rectangular ducts use $D_i = \sqrt{\frac{4(Height)(Width)}{\pi}}$

ΔP - Differential pressure (inches of water column)

DP - Differential pressure ($\frac{lb_f}{ft^2}$)

g - Local gravitational acceleration rate ($\frac{ft}{s^2}$)

g_c - Gravitational constant of proportionality ($\frac{32.174049\ lb_m \cdot ft}{lb_f \cdot sec^2}$)

h - height of fluid pressure column (ft)

K - Flow coefficient

k = Isentropic exponent - ratio of specific heat at constant pressure to that at constant volume

M - Mass flow rate ($\frac{lb_m}{hr}$)

P - Pressure ($\frac{lb_f}{in^2}$) absolute PSIA

P_f - Pressure ($\frac{lb_f}{ft^2}$) absolute

Q - Fluid flow in cubic feet per second (CFS)

Q_a - Gas flow in actual cubic feet per minute (ACFM) for gas

Q_L - Liquid flow in gallons per minute (GPM)

Q_p - Gas flow in pound per hour (PPH) (used extensively with steam flow)

Q_s - Gas flow in standard cubic feet per minute (SCFM) for gas.

S - Specific gravity of the flowing fluid. For liquids S=1 for water at 60°F 14.6959 PSIA. For gases S=1 for dry air at 60°F, 14.6959 PSIA)

T - Temperature (degrees Rankine) T=t+459.67

t - Temperature (degrees Fahrenheit)

V - Velocity ($\frac{ft}{s}$)

Z - Elevation (ft)

ρ - Density ($\frac{lb_m}{ft^3}$)